SCIENCE FOR GLASS PRODUCTION

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PREDICTION OF SHEET GLASS BEHAVIOR ON BENDING

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A method for calculating glass sheet deformation based on a general model at temperatures above the vitrification temperature is proposed and allows analytical determination of design parameters of molding equipment elements and their operating conditions.

Relaxation of deformations and stresses is extremely significant for glass rheology. In particular, S. M. Rekhson and V. A. Ginzburg in their studies [1] discuss a model describing the relaxation of the glass structure, stresses and deformations (IKhS model) which clarifies the character of modifications of the parameters mentioned above within the vitrification interval. The model uses an arbitrarily set history of variation of stress σ and deformation δ in time τ. According to [1], the total deformation is described by the following expression

$$\varepsilon(\tau) = \varepsilon_d(\tau) + \varepsilon_\eta(\tau) + \varepsilon(0), \tag{1}$$

where $\varepsilon_d(\tau)$ is the delayed elastic component of deformation; $\varepsilon_\eta(\tau)$ is the viscous component of deformation; $\varepsilon(0)$ is the instantaneous (elastic) deformation component.

A graphic interpretation of these types of deformation is presented in Fig. 1.

According to the theory of solids, delayed elastic deformation is similar to creep.

In calculation of deformation based on Eq. (1), the most complicated is determination of $\varepsilon_d(\tau)$, while the other components can be easily calculated from the formulas

$$\varepsilon_h(\tau) = \sigma(0)\frac{\tau}{h};$$

$$\varepsilon(0) = \frac{\sigma(0)}{E},$$

where $\sigma(0)$ is the initial stress under loading; τ is the loading or relaxation time; η is the glass viscosity E is the modulus of elasticity in tension.

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In the general case, according to the data in [1], the creep component of deformation is found from the expression

$$\varepsilon_d(\tau) = \varepsilon_d(\infty)(1 - M_\varepsilon(\tau)), \tag{2}$$

where $\varepsilon_d(\infty)$ is magnitude of the function $\varepsilon_d(\tau)$ at $\tau \to \infty$; $M_\varepsilon(\tau)$ is the deformation relaxation function.

The magnitude of the function M* is calculated from the following formula derived from the recommendations in [1 – 3].

$$M_\varepsilon(\tau) = \exp(-(\tau/\tau_r)^b),$$

where τ_r is the relaxation time; b is a constant ($b = 0.5$)

In Eq. (2), only parameter $\varepsilon_d(\infty)$ is unknown. The method for determination of this component is based on the geometric decomposition of function $\varepsilon(\tau)$ into its components (Fig. 2).

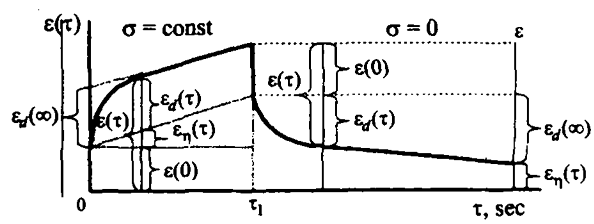

Fig. 1. Diagram of three types of deformation.

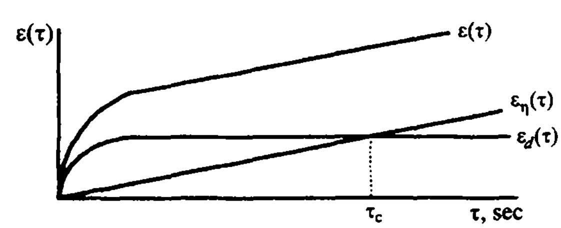

Fig. 2. Dependence of the total deformation on its components.

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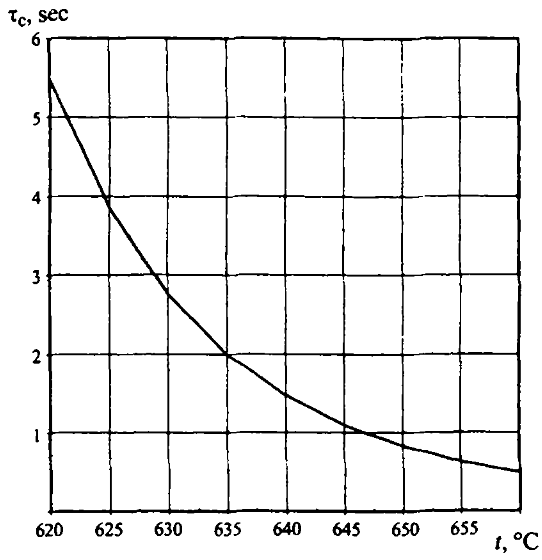

Fig. 3. Temperature dependence of τ_c.

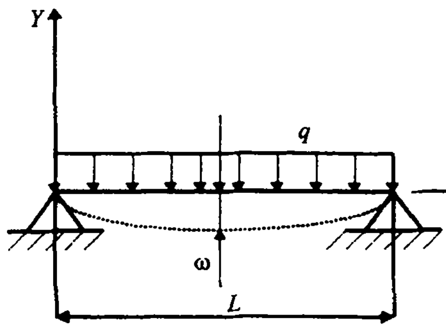

Fig. 4. Design diagram for determining $\omega(\varepsilon)$ under loading with a distributed load.

According to Eq. (2), function $\varepsilon_d(\tau)$ approaches a certain value $\varepsilon_d(\tau)_{max}$ which is nothing else but $\varepsilon_d(\infty)$. Thus, it can be postulated that as soon as $\varepsilon_d(\tau)$ reaches a certain value, function $\varepsilon(\tau)$ depends only on the viscous component $\varepsilon_\eta(\tau)$, and the contribution of $\varepsilon_d(\tau)$ to the total deformation is limited by a certain constant value. The characteristic point in this case is $\varepsilon(\tau_c)$ at which, at time τ_c, the values of components $\varepsilon_\eta(\tau)$ and $\varepsilon_d(\tau)$ become equal.

Assuming that for constant temperature and loading conditions $\varepsilon_d(\infty)$ is a constant value, τ_c can be found, provided that the value of the function

$$f(\tau) = 1 - M_\varepsilon(\tau)$$

for this point is known. This value is determined experimentally (Fig. 3). Then, denoting the value of $f(\tau)$ as φ we can write:

$$\tau_c = \tau_r(-\ln(1-\varphi))^{1/b}.$$

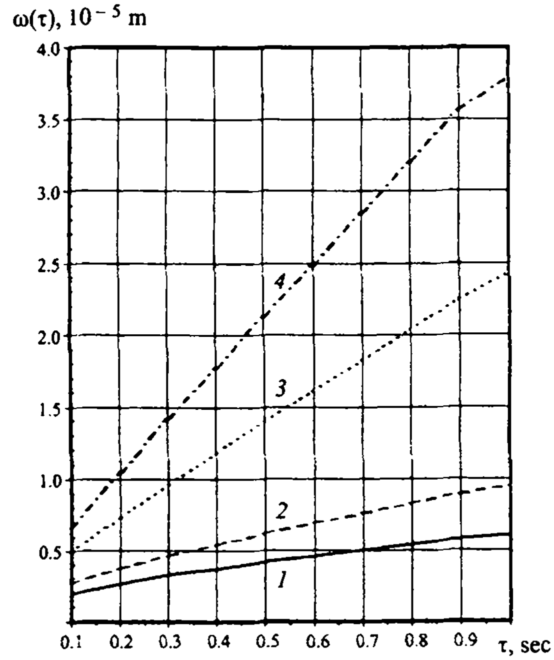

Fig. 5. Variation of sample deflection depending on temperature: *1*, *2*, *3*, and *4*) 620, 630, 650, and 660°C, respectively.

Using the value obtained, $\varepsilon_d(\infty)$ can be determined:

$$\varepsilon_d(\infty) = \frac{\sigma(0)\tau_c}{\eta(1 - M_\varepsilon(\tau_c))}.$$

Since all components of function $\varepsilon_d(\tau)$ are found, total deformation $\varepsilon(\tau)$ can be determined as well.

Using the expression obtained for total deformation $\varepsilon(\tau)$, one can predict the behavior of a glass sample both in deformation and in relaxation of deformation, which is the most important moment of sheet glass deformation.

Consider the application of the given model to bending deformation. The loading diagram is shown in Fig. 4. For this loading scheme, the equation of the bent beam axis can be described by equation [4]

$$\omega(Z) = -\frac{qL^4}{24E_2J}\left(\frac{Z}{L} - 2\left(\frac{Z}{L}\right)^3 + \left(\frac{Z}{L}\right)^4\right),$$

where q is the distributed load; L is the beam length; E_2 is the shear modulus; J is the moment of inertia of the beam lateral section; Z is a longitudinal coordinate.

Then, using the expression

$$\frac{1}{\rho} = \frac{d^2y}{dx^2} = \frac{M(Z)_{max}}{E_2J};$$

$$\varepsilon_{max} = \frac{h}{2}\frac{1}{\rho},$$

where ρ is the radius of curvature of the beam middle layer; $M(Z)_{max}$ is the maximum torque; h is the beam thickness, one can obtain the expression for $\omega(\varepsilon)$:

$$\omega(\varepsilon) = \frac{5L^2\varepsilon(\tau)}{24h}.$$

Fig. 5 shows the variation in the deflection $\omega(\tau)$ of the sample depending on the temperature at the loading site. Relaxation curves are constructed and calculated with the same formulas taking into account relaxation of stress in the holding time segment. These curves were obtained on the basis of the model considered and the sample loading diagram is shown in Fig. 4. The calculations were carried out for the following loading procedure: loading of the sample, holding time (relaxation of stress), removal of the load (relaxation of deformations). The following initial data were used: deformation time of 1 sec; beam length of 0.1 m; beam thickness of 0.004 m.

On the plots in Fig. 5, the elastic component is absent. That was done for more accurate rendering of the process. This particular component can be determined by the respective formula.

Based on the curves obtained, the time and temperature parameters of the glass sample deformation can be determined for prescribed limiting values of residual stress deformation relaxation.

The method considered will become the basis for determining the design parameters of molding equipment elements and their operating conditions.

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